9.1, 9.2, and 9.3 Multivariable systems

System of Equations:
a set of open sentences containing the same variables

Solution Set:

the set of all ordered pairs that satisfy all open sentences in the system.

Consistent: a system of equations with at least one solution.

-Either independent or dependent

Independent: has one solution

$$3x + 4y = 8$$
$$2x + 3y = 9$$

<u>Dependent:</u> has infinitely many solutions

$$18x + 12y = -24$$
$$12x + 8y = -16$$

$$0 = 0$$

$$\boxed{\left(-\tfrac{2}{3}a - \tfrac{4}{3},\ a\right)}$$

$$12x + 8a = -16$$
$$12x = -8a - 16$$
$$x = -\frac{2}{3}a - \frac{4}{3}$$

Inconsistent: a system of equations with NO solution

$$15x + 12y = 24$$
$$10x + 8y = 8$$

$\emptyset$ sol.

$$0 = 3$$

$$
\begin{aligned}
x + y + z &= 6 \\
2x - y + z &= 3 \\
3x \qquad - z &= 0
\end{aligned}
$$

What does the solution look like?

Solve:

$3\times 3 \to 2\times 2 \to 1\times 1$

$$\begin{aligned} x - 2y + 3z &= 9 \\ -x + 3y &= -4 \\ 2x - 5y + 5z &= 17 \end{aligned}$$

Goal is to get it to the following form:

$$\begin{aligned} x - 2y + 3z &= 9 \\ y + 3z &= 5 \\ z &= 2 \end{aligned}$$

This is called "row-echelon" form

Gaussian elimination

-Invented by Carl Friedrich Gauss (1777-1855)

-It's the process of using elementary row operations to convert a system of equations into row echelon form $3 \times 3 \to 2 \times 2 \to 1 \times 1$

Elementary Row operations:

1) interchange two rows
2) multiply (or divide) one of the equations by a nonzero constant
3)Add a multiple of one equation to another equation

$$\begin{aligned} x + y + z &= 6 \\ 2x - y + z &= 3 \\ 3x \quad\quad - z &= 0 \end{aligned}$$

R3 $\quad 3x - z = 0$

R1+R2 $\quad 3x + 2z = 9$

R1-R2 $\quad -3z = -9$

$z = 3$

Back substitution

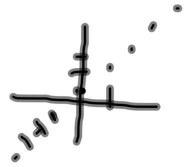
$3x - 3 = 0$

$x = 1$

$1 + y + 3 = 6$

$y = 2$

$(1, 2, 3)$



$$\begin{aligned} 2x + y - 3z &= 4 \\ 4x \quad + 2z &= 10 \\ -2x + 3y - 13z &= -8 \end{aligned}$$

$4x + 2z = 10$

$-3R1+R3$ $\quad -8x - 4z = -20$

$0 = 0$

$\left(-\frac{1}{2}a + \frac{5}{2},\ 4a - 1,\ a\right)$

$-6x - 3y + 9z = -12$

$-2x + 3y - 13z = -8$

$-8x - 4z = -20$

$4x + 2a = 10$

$4x = -2a + 10$

$x = -\frac{1}{2}a + \frac{5}{2}$

$2\left(-\frac{1}{2}a + \frac{5}{2}\right) + y - 3a = 4$

$-a + 5 + y - 3a = 4$

$y = 4a - 1$

$$\begin{aligned} 2\quad x - 3y + z &= 1 \\ 2x - y - 2z &= 2 \\ x + 2y - 3z &= -1 \end{aligned}$$

$2R1+R2$ $\quad 4x-7y=4$

$3R1+R3$ $\quad 4x-7y=2$

$0=-2$

$\emptyset$

Find a quadratic equation $y = ax^2+bx+c$
whose graph passes through the points
(-1, 3), (1, 1), and (2,6)

$3 = a - b + c$

$1 = a + b + c$

$6 = 4a + 2b + c$

* now solve 3x3

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